SEM – VI

LINEAR HARMONIC OSCILLATOR →

A linear harmonic oscillator is a system in which the restoring force is directly proportional to displacement and acts in the opposite direction

Restoring force

$$F = -Kx$$

K = Force constant

x = displacement from equilibrium

Equation of motion

using Newtons second law

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega = \sqrt{\frac{K}{m}}$

Solution of the Equation

$$x(t) = A\sin(\omega t + \phi)$$

$$x(t) = A\cos(\omega t + \phi)$$

where A = Amplitude

ϕ = Phase constant

velocity and acceleration

velocity

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

Maximum velocity

$$V_{max} = A\omega$$

Acceleration

$$a = \frac{d^2x}{dt^2} = -\omega^2$$

Maximum acceleration

$$a_{max} = \omega^2 A$$

Time Period and Frequency

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

Energy of Linear Harmonic oscillator

Kinetic energy

$$K.E = \frac{1}{2}mv^2$$

Potential energy

$$P.E = \frac{1}{2}kx^2$$

Total energy

$$E = K.E + P.E = \frac{1}{2}kA^2$$

Total energy remains constant

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